Department of Civil and Environmental Engineering

Examination paper for TVM4155 Numerical modelling and hydraulics

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Examination time (from-to): 09:00-13:00

Permitted examination support material: Code D. No printed or hand-written support material is allowed. A specific basic calculator is allowed:

Casio fx-82ES PLUS and Casio fx-82EX

Citizen SR-270X and Citizen SR-270X College

Hewlett Packard HP30S

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Informasjon om trykking av eksamensoppgave

Originalen er:

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Problem 1

Given the following equation for the concentration of sediments in a river:

$$\frac{\partial c}{\partial t}+U_j\frac{\partial c}{\partial x_j}+w\frac{\partial c}{\partial x_3}=\frac{\partial}{\partial x_j}(\Gamma\frac{\partial c}{\partial x_j})+S$$

a) Describe all terms in the equation.

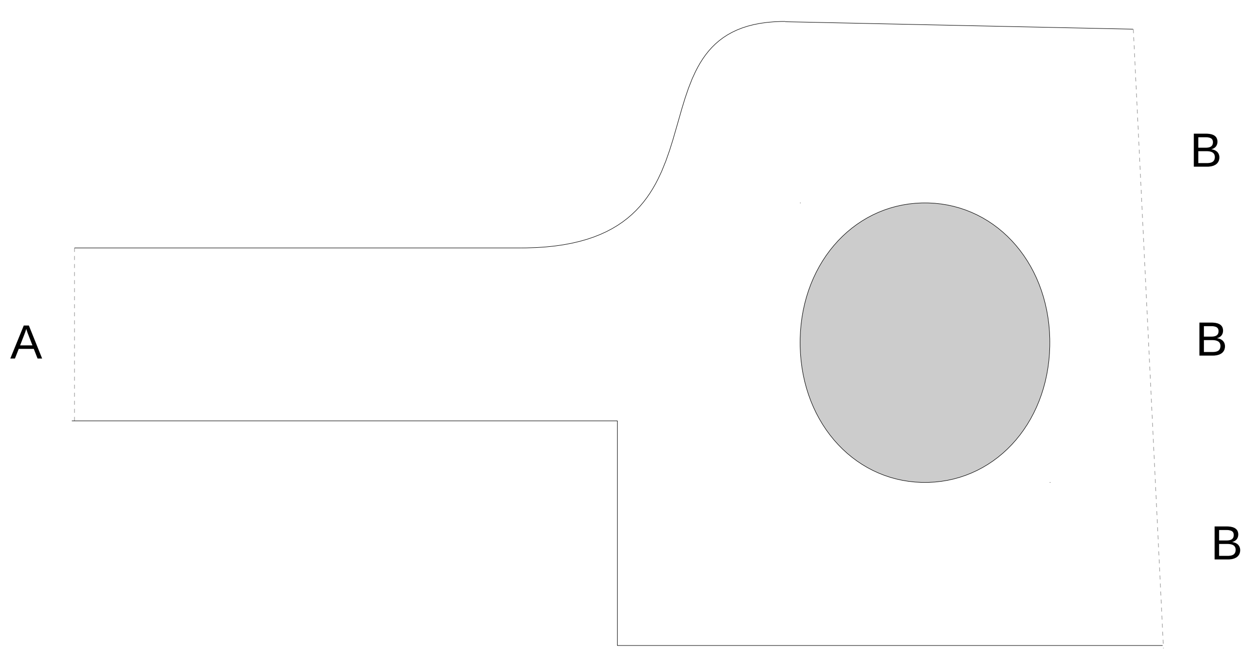
b) How is erosion of sediments taken into account in the formula?

c) Derive formulas for how the convective and diffusive terms can be discretized using a first-order upwind approach

d) Derive the formula for how the time term is discretized

Problem 2

Make a structured two-dimensional grid with quadrilateral cells for the geometry given below. Use between 150 and 300 cells. The inflow is along the A side. The outflow side is marked with B. The flow is from left to right. The circle is an obstacle where the fluid can not flow. You may use outblocking. You may draw on the exam paper and hand this in.



Problem 3

Water is discharged vertically up from the bottom of a lake which is 20 meters deep. The water in the lake has a temperature of 5 degrees Centergrade, while the water coming from the pipe has a temperature of 20 degrees Centergrade. The pipe diameter is 1 meter and the discharge of water from the pipe is 50 l/s.

a) What is the water velocity 10 meters directly over the pipe?
b) What is the water velocity 14 meters over the pipe, but 2 meters to the side of the pipe?
c) What is the discharge of water in the plume 8 meters above the lake bed?
d) What is the reason why the plume discharge changes above the lake bed?

Problem 4

a) What is a particle Reynolds number?
b) Describe the research Shields did with regards to erosion.
c) How is the particle Reynolds number used in determining erosion of a particle?
d) If the river bed has a considerable lateral slope, how can we take this into account when computing erosion?
e) If the river bed consists of some coarse and some fine particles, how does this affect the erosion of the particles?
f) Which formulas can be used in question e) ?

Problem 5

Olsen and Skoglund (1994) studied the flow in a sand trap and compared the results with measurements from a physical model study.

a) Describe the measurement techniques used in the laboratory

b) Describe at least four features of the grid that was used

c) Describe the sand/silt that was modelled, and how the non-homogenous sediment size was taken into account in the numerical model

d) Which turbulence model was used, and how was it modified in the study?

e) How sensitive was the total sediment trap efficiency for the modifications of the turbulence model?

Problem 6

Given the equation below:

$$\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}+g\frac{\partial y}{\partial x}-g(I_b-I_f)=0$$

a) What is the equation called and what is it used for? Which programs are used to solve it?

b) Describe each term in the equation

c) Derive the equation

Problem 7

The 10th of October 2016, a Dutch tanker missed the shipping lights in the Rhine river in Germany and got stuck on the Jungferngrund sand bar (see photo), near St. Goar. The tanker contained 786 tons of a toxic chemical. Luckily, no chemical was spilled, as the soft sand did not do any damage to the vessel. Let us assume that the ship had hit a rock outcrop 300 meters further downstream instead, and all the chemicals had spilled into the river over a time period of 10 minutes. What would the maximum chemical concentration be at Koblenz, 44 kilometers downstream? The discharge in the Rhine was 500 m^3/s, and the average water depth was 1.0 meters. The slope of the river in this reach is 1/4000, and the average width is 300 meters.

Tables and formulas

$$\frac{u}{u_{max}} = \left(1+\frac{r^2}{0.016}x\right)^{-2} \qquad \frac{u_{max}}{u_0} = 6.4\left(\frac{x}{d_0}\right)^{-1} \qquad \frac{u}{u_0} = 4.3\,Fr'^{\frac{-2}{3}}\left(\frac{z}{d_0}\right)^{\frac{-1}{3}} e^{\left[-96\frac{r^2}{z^2}\right]}$$

$$\frac{Q}{Q_0} = 0.42\frac{x}{d_0} \qquad \frac{Q}{Q_0} = 0.18\,Fr'^{-2/3}\left(\frac{z}{d_0}\right)^{5/3}$$

Temperature (C)	Density (kg/m³)		Atom	Weight
0	999.87		Hydrogen	1
2	999.97		Carbon	12
4	1000		Nitrogen	14
6	999.97		Oxygen	16
8	999.88			
10	999.73			
12	999.52			
14	999.27			
16	998.97			
18	998.62			
20	998.23			
22	997.80			
24	997.33			
26	996.81			

$$\Gamma = 0.058\frac{Q}{IB} \qquad \Gamma = 0.11\frac{(UB)^2}{Hu_*}$$

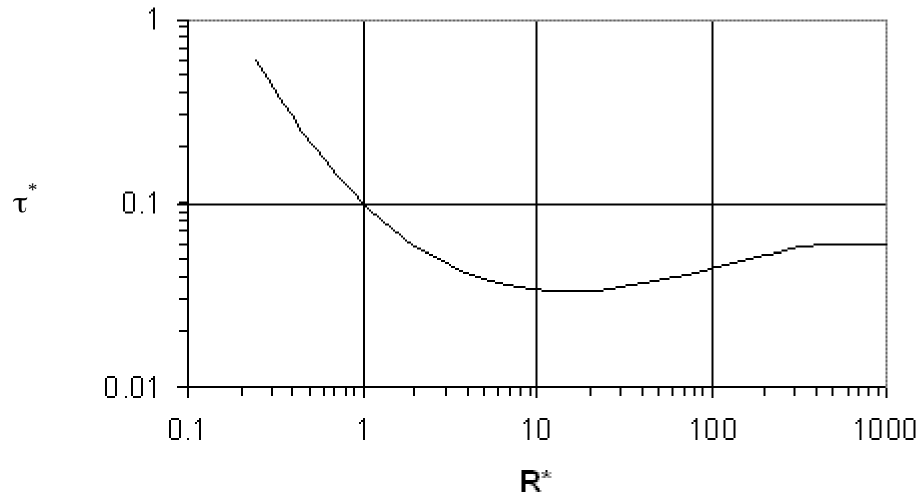
$$c(x,t) = \frac{c_0 L}{2\sqrt{(\pi\Gamma t)}} e^{-\frac{(x-Ut)^2}{4\Gamma t}} \qquad Fr' = \frac{u_0}{\sqrt{\left(\frac{\rho_{res}-\rho_0}{\rho_{res}}\right) g\, d_0}}$$

$$\frac{\rho-\rho}{\rho} = 9\,Fr'^{-\frac{2}{3}}\left(\frac{z}{d_0}\right)^{\frac{5}{3}} e^{\left[-71\frac{r^2}{z^2}\right]}$$

$$U = 1/n\, r_h^{2/3} I^{1/2} \qquad U = C\, r_h^{1/2} I^{1/2}$$

$$\rho_s = 2650\, kg/m^3 \qquad M = \frac{26}{d_{90}^{1/6}} \qquad \tau = \rho g h I \qquad u_* = \sqrt{\left(\frac{\tau}{\rho}\right)}$$

$$\nu = 10^{-6} m^2/s$$



$$R^* = u_* \frac{d}{\nu} \qquad \tau^* = \frac{\tau}{d(\rho_s - \rho_w) g}$$

$$\frac{dy}{dx} = \frac{I_f - I_0}{1 - Fr^2} \qquad q_s = \frac{1}{g}\left[\frac{\rho_w g r_h I - 0.047 g (\rho_s - \rho_w) d_{50}}{0.25 \rho_w^{1/3} \left(\frac{\rho_s - \rho_w}{\rho_s}\right)^{\frac{2}{3}}}\right]^{\frac{3}{2}}$$

$$q_s = 0.025 \rho_s U^2 \sqrt{\frac{d_{50}}{g\left(\frac{\rho_s}{\rho_w} - 1\right)}} \left[\frac{\tau}{g(\rho_s - \rho_w) d_{50}}\right]^{\frac{3}{2}}$$

$$K = \frac{-\sin\phi \sin\alpha}{\tan\theta} + \sqrt{\left(\frac{\sin\phi\sin\alpha}{\tan\theta}\right)^2 + \cos^2\phi \left[1 - \left(\frac{\tan\phi}{\tan\theta}\right)^2\right]} \qquad \xi_i = \left(\frac{d_i}{d_{50}}\right)^{-0.3}$$

$$K = 0.954 \left(1 - \frac{\phi}{\theta}\right)^{0.745} \left(1 - \frac{\alpha}{\theta}\right)^{0.372}$$

Solution:

Problem 1:

a) - Time term: how the concentration changes over time
- Convective term, how the sediments are transported with the average water velocity
- Fall velocity term: how the fall velocity of the particles affects the concentration
- Diffusive term: how the turbulent diffusion affects the concentration
- Source term: For example from boundary conditions

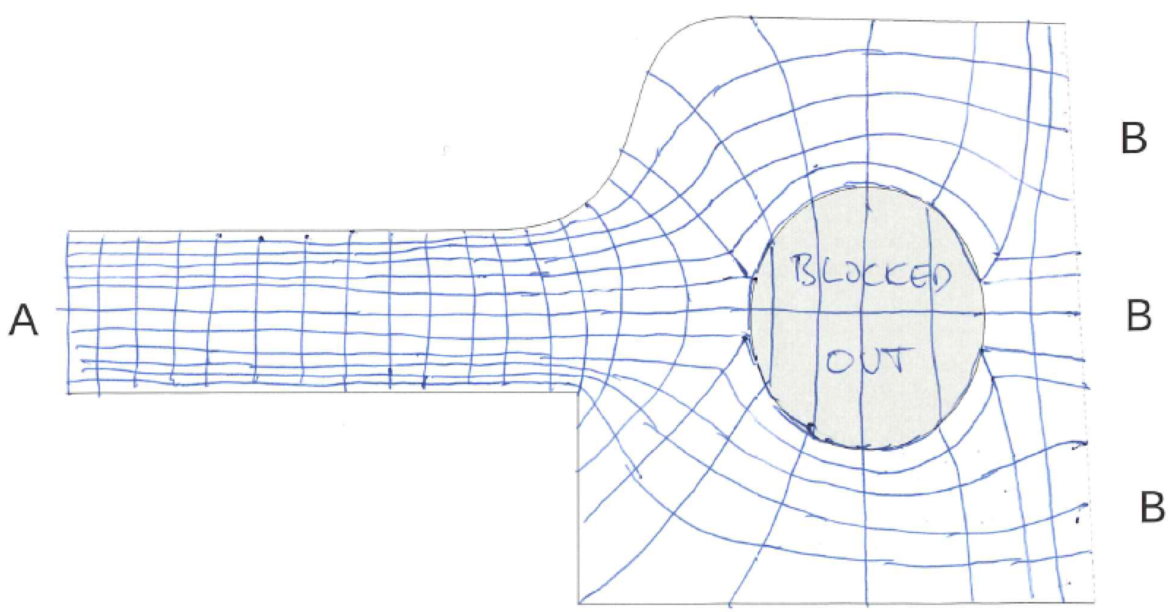
b) Erosion is modelled from a positive source term. Or a concentration given by a formula is used in the bed cells, instead of solving the convection-diffusion equation.

c) See Chapter 5.3 in the class notes.

d) See Chapter 5.7 in the class notes.

Problem 2:

12×23 = 276 CELLS



A misprint unfortunately occurred on the exam paper, so the lower line was changed to be horizontal.

Problem 3

Given values:

- Temperature lake: $T_0 = 5\ °$
- Temperature plume: $T_{res} = 20\ °C$
- Pipe diameter: $d_0 = 1{,}0\ m$
- Discharge: $Q = 0{,}050\ m^3/s$
- Lake depth: $H = 20\ m$

a) Seeing as how the temperature in the lake is different from water exiting the pipe, we have to find the densimetric Froude number:

$$F_r` = \frac{u_o}{\sqrt{\left(\frac{\rho_{res} - \rho_0}{\rho_{res}}\right) g d_0}}$$

We need to find the velocity of the plume when exiting the pipe:

$$u_0 = \frac{Q}{A_0} = \frac{0{,}050\,\frac{m^3}{s}}{\frac{1}{4}\pi(1{,}0\ m)^2} = 0{,}064\,\frac{m}{s}$$

We can then calculate Fr`:

$$F_r` = \frac{0{,}064\,\frac{m}{s}}{\sqrt{\left(\frac{999{,}99 - 998{,}23}{999{,}99}\right) \cdot 9{,}81\,\frac{m}{s^2} \cdot 1{,}0\ m}} = 0{,}484$$

Finally, we can find the velocity 10 above the pipe:

$$\frac{u}{u_0} = 4{,}3\ F_r`^{-\frac{2}{3}} \left(\frac{z}{d_0}\right)^{-\frac{1}{3}} e^{\left[-96\frac{r^2}{z^2}\right]}$$

$$\frac{u}{u_0} = 4{,}3 \cdot 0{,}484^{-\frac{2}{3}} \cdot \left(\frac{10\ m}{1{,}0\ m}\right)^{-\frac{1}{3}} e^{\left[-96 \cdot \left(\frac{0}{10\ m}\right)^2\right]} = 3{,}24$$

$$u = 3{,}238\ u_0 = 0{,}21\,\frac{m}{s}$$

b) Apply the same procedure with z = 14 m and r = 2 m:

$$\frac{u}{u_0} = 4{,}3 \cdot 0{,}484^{-\frac{2}{3}} \cdot \left(\frac{14\ m}{1{,}0\ m}\right)^{-\frac{1}{3}} e^{\left[-96 \cdot \left(\frac{2\ m}{14\ m}\right)^2\right]} = 0{,}408$$

$$u = 3{,}203 \cdot 10^{-6}\ u_0 = 0{,}026\,\frac{m}{s}$$

c) Finding the discharge z = 8 m over the pipe exit:

$$\frac{Q}{Q_0} = 0{,}18\, F_r^{-\frac{2}{3}} \left(\frac{z}{d_0}\right)^{\frac{5}{3}} = 0{,}18 \cdot 0{,}484^{-\frac{2}{3}} \cdot \left(\frac{8\,m}{1{,}0\,m}\right)^{\frac{5}{3}} = 9{,}344$$

$$Q = 9{,}344\, Q_0 = 0{,}467 \frac{m^3}{s}$$

d) Because the surrounding water is mixed into the plume, the discharge will increase with the distance from the outlet.

Problem 4

a) A number that describes the liquid flow past a particle. It is related to describing the drag and fall velocity of the particle through a certain liquid.

$$R^* = u_* \frac{d}{\nu}$$

The difference between Reynolds number and the particle Reynolds number, is the shear velocity u_* and the diameter of the particle d.

b) Shields researched the critical shear stress for movement of sediment particles of different sizes.

c) It is used via Shields diagram to determine the critical shear stress of a sediment particle with given size and density.

d) A considerable lateral slope can be taken into account by using the following formula:

$$K = -\frac{\sin\phi \sin\alpha}{\tan\theta} + \sqrt{\left(\frac{\sin\phi \sin\alpha}{\sin\theta}\right)^2 + \cos^2\phi \left[1 - \left(\frac{\tan\phi}{\tan\theta}\right)^2\right]}$$

Parameter	Description
K	Decrease in critical shear stress
α	The angle between the directions of the flow and the channel
ϕ	Angle of the slope
θ	Slope parameter

The factor K is multiplied with the critical shear stress to give the effective shear stress. K is between 0 and 1.

e) Coarse and large particles in a river bed acts as protective barriers, which smaller erodible particles can hide behind and be stabilized by. Consequently, this reduces the erosion of the riverbed. However, the coarser and larger particles will protrude above the riverbed and will be more unstable.

f) The parameter:

$$\xi_i = \left(\frac{d_i}{d_{50}}\right)^{-0.3}$$

Is multiplied with the Shields factor.

Problem 5

a) Electronic current meter to measure velocities. Sediment concentrations from water samples taken through brass pipes.

b) Grid: Structured, non-orthogonal, hexahedral cells, three-dimensional.

c) Four sediment sizes: 0.1 to 0.4 mm. Fall velocities 0.8-7 cm/s. Four convection-diffusion equations were solved.

d) K-epsilon model. Modification: σ epsilon changed from 1.3 go 0.9.

e) Trap efficiency was 87.1 % for the standard k-epsilon model and 88.3 % for the modified model.

Problem 6

a) The given equation is the Saint - Venant equation.

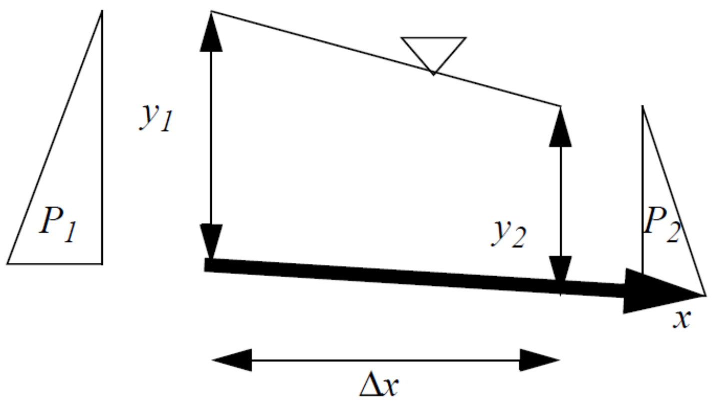
$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} + g\frac{\partial y}{\partial x} - g(I_0 - I_f) = 0$$

The equation is used for modelling unsteady open-channel flow. It is typically used for dam-break modelling and flood wave modelling. It is a 1D equation. Examples of programs used to solve the Saint – Venant equations are HEC-RAS, DAMBRK, MIKE11 and ISIS.

b) Description of the terms:

Term	Description
$\frac{\partial U}{\partial t}$	Acceleration term
$U\frac{\partial U}{\partial x}$	Momentum term
$g\frac{\partial y}{\partial x}$	Pressure term
gI_0	Gravity term
gI_f	Bed shear term

c) Taking basis in the following control volume.



The Saint – Venant equation is derived from Newton`s second law:

$$\sum F = ma$$

The acceleration term on the right side can be written as:

$$ma = m\frac{dU}{dt} = \rho V\frac{dU}{dt} = \rho y B\Delta x\frac{dU}{dt}$$

The external forces on the volume are gravital forces, bed shear stress, pressure forces and momentum forces:

$$\sum F = F_g + F_b + F_p + F_m$$

Gravitational component in the x - direction:

$$F_g = mg\sin\alpha = \rho V g\sin\alpha = \rho g y B\Delta x\sin\alpha \approx \rho g y B\Delta x I_b$$

Bed sheer stress:

$$F_b = -\tau A = \tau B\Delta x$$

F_b is negative because the friction force is in the negative x – direction. Often an energy slope is introduced:

$$I_f = \frac{\tau}{\rho g y}$$

$$F_b = -\rho g y B\Delta x I_f$$

Pressure forces:

Two pressure forces acts on the control volume, one on each side. The total force from the pressure gradient is the sum of the hydrostatic pressures.

$$F_p = P_1A_1 - P_2A_2 = \frac{1}{2}\rho g B y_1^2 - \frac{1}{2}\rho g B y_2^2 = \frac{1}{2}\rho g B(y_1^2 - y_2^2)$$

$$F_p = \frac{1}{2}\rho g B\left(y^2 - \left(y + \frac{dy}{dx}\Delta x\right)^2\right)$$

$$F_p = \frac{1}{2}\rho g B\left[y^2 - y^2 - \left(2\frac{dy}{dx}y\Delta x\right) + \left(\frac{dy}{dx}\Delta x\right)^2\right]$$

The last term is negligible:

$$F_b = -\rho g B y\frac{dy}{dx}\Delta x$$

Momentum equation:

$$F_m = \rho Q(U_1 - U_2) = \rho U B y\left(U - \left(U + \frac{dU}{dx}\Delta x\right)\right)$$

$$F_m = -\rho U B y\frac{dU}{dx}\Delta x$$

If we put in the all the forces in Newton`s second law, where we started:

$$\sum F = F_g + F_b + F_p + F_m = ma$$

All of the terms have the following common factors: ρ, B, y and Δx.
The equation can then be simplified to:

$$gI_b - gI_f - g\frac{dy}{dx} - U\frac{dU}{dx} = \frac{dU}{dt}$$

$$\frac{dU}{dt} + U\frac{dU}{dx} + g\frac{dy}{dx} - g(I_b - I_f) = 0$$

Problem 7:

Given values:

- Mass of chemical:	m_c = 786 000 kg
- Spill time:	T_s = 10 min
- Distance Koblenz:	x = 44 km
- Discharge:	Q = 500 m^3/s
- Water depth:	y = 1,0 m
- Bed slope:	I_0 = 1/4000
- River width:	B = 300 m

Assuming constant velocity and diffusion, we can use the following analytical solution for the convection – diffusion equation:

$$c(x,t) = \frac{c_0 L}{2\sqrt{\pi \Gamma t}} e^{-\frac{(x-Ut)^2}{4\Gamma t}}$$

We need to find the initial concentration c_0, the dispersion constant Γ, the velocity U and the spill length L:

$$C_0 = \frac{m_c}{QT_s} = \frac{786\,000\,kg}{500\frac{m^3}{s} \cdot 10 \cdot 60\,s} = 2{,}62\frac{kg}{m^3} = 2620\,ppm$$

$$U = \frac{Q}{By} = \frac{500\frac{m^3}{s}}{300\,m \cdot 1{,}0\,m} = 1{,}67\frac{m}{s}$$

$$\Gamma = 0{,}058\frac{Q}{IB} = 0{,}058 \cdot \frac{500}{\frac{1}{4000} \cdot 300} = 386{,}67$$

$$L = UT_s = 1{,}67\frac{m}{s} \cdot 10 \cdot 60\ s = 1000\ m$$

We can now set up the equation for the concentration at Koblenz:

$$c(44\,000, t) = \frac{2620 \cdot 1000}{2\sqrt{\pi \cdot 386{,}67 \cdot t}} e^{-\frac{(44000 - 1{,}67t)^2}{4 \cdot 386{,}67 \cdot t}}$$

The maximum concentration at Koblenz will happen close to:

$$t = \frac{x}{U} = \frac{44000\ m}{1{,}67\frac{m}{s}} = 26400\ s$$

t (s)	c(t) [ppm]
26000	230,5
26100	231,2
26200	231,6
26300	231,6
26400	231,3
26500	230,7
26600	229,8
26700	228,6
26800	227,1
26900	225,4

The maximum chemical concentration at Koblenz would be approx. 232 ppm.