Problem 1.

A river is considered wide and has a discharge of 2 m^2/s. The bed slope is 1:300, and the Manning -Strickler number is 60.

a) Compute the water depth in the river for uniform flow.

b) Compute the sediment transport with a total load formula, if the average particle size is 1 mm. The flow is still uniform.

c) Which equation is used to compute steady 1D non-uniform water surface profiles? Write the equation.

d) The river is dammed by a small reservoir. The water depth just upstream of the reservoir is 2 meters. Compute the water depth 300 meters upstream of the dam. You only need to use one cross-section in addition to the cross-section at the dam. If you need to iterate, you can start by guessing dy is -1 m.

Problem 2.

a) The kinetics of biochemical reactions can follow equations with different orders. Of which order is the equation that describes the kinetics of most biochemical reactions in water? Write the equation.

b) Balance the following equation

$H_2O + CO_2 \text{ -> } O_2 + C_6H_{12}O_6$

c) 100 grams CO_2 is used in a photosynthesis. How many grams of oxygen is produced?

d) Describe the mechanism where there will be large spatial variation of concentrations of phyto-plankton in a lake. How can the wind direction compared with the concentration gradient tell us something about the phytoplankton species?

Problem 3.

You work for a consulting company, and you are asked to use a CFD program to compute the coefficient of discharge for three new spillways that has not yet been built. The spillways are slightly different from standard spillways. The grid is three-dimensional.

a) Which are the most important errors and uncertainties in your computations? What should you do to evaluate these errors? Will a high-order discretization scheme have false diffusion? Why?

b) You do grid independency tests for the three spillways, which are named Alpha, Beta and Delta and get the answers in the table below. The first grid gives a coefficient of discharge denoted C_1, and the second grid gives a coefficient of discharge denoted C_2. The computer program uses a first-order upwind method for the discretization, which means that p is 1.0 in the GCI formula.

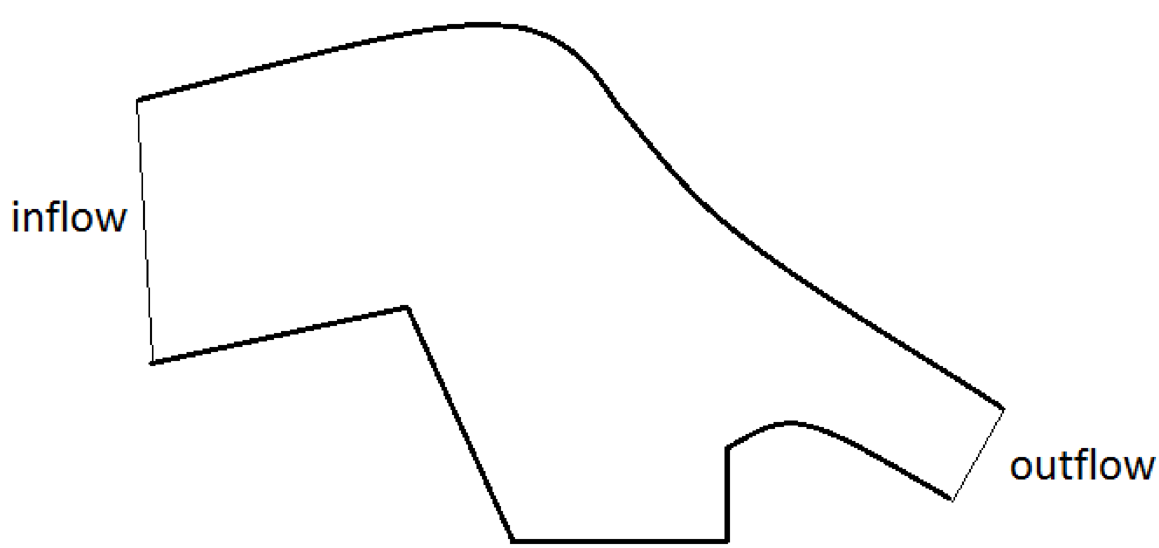
Table 1: Number of cells in the grids and coefficients of discharge (C) for the parameter sensitivity tests

Spillway	1st grid size	C_1	2nd grid size	C_2
Alpha	1 000 000	1.70	8 000 000	1.73
Beta	1 000 000	1.75	2 000 000	1.72
Delta	1 000 000	1.80	1 100 000	1.81

Compute the error due to the grid resolution according to the GCI method for each of the three spillways.

Problem 4

Make a structured 2D grid for the geometry in the figure below. Only use quadrilateral cells. Use between 100 and 200 cells. Do not use outblocking. The thinner lines are inflow/outflow regions and the thicker lines are side walls, as given by the text in the figure. You may make the drawing on this paper and hand it in.



Problem 5.

A lake has a square shape seen from above, and it is 2 km in the north-south direction and 1 km in the east-west direction. The depth of the lake is 100 meters. The wind speed is 20 m/s, 10 meters above the lake.

a) Compute the slope of the water surface at the lake in the north-south direction.

b) A thermocline is located at 8 meters depth. Compute the slope of the thermocline. The temperature is on average 6 degrees Centergrade below the thermocline and 14 degrees Centergrade above the thermocline.

c) Derive the equation for the slope of the water surface of a lake, given the shear stress from the wind. This is the same equation you used in problem a).

Problem 6.

The article by Hillebrand, Klassen and Olsen describes the computation of sediment transport in the Iffezheim reservoir, on the German/French border.

a) Which equations were solved to compute the sediment transport?

b) The study computed the sediment deposition in the reservoir over a time period, and compared this with sediment deposition measurements over the same time period. How long was this time period? And how long was the computational time, and which computers were used?

c) How was it possible to simulate such a long time period in such a short computational time?

d) A parameter sensitivity test was done. Which parameters were varied?

e) Which parameters had most effects on the results?

Formulas and tables:

$$U = \frac{1}{n} r_h^{\frac{2}{3}} I^{\frac{1}{2}} \qquad r_h = \frac{A}{P} \qquad M = \frac{1}{n} \qquad M = \frac{26}{d_{90}^{\frac{1}{6}}}$$

$$a = -fU = -\left(\frac{4\pi}{T}\sin\phi\right)U, \qquad f\text{=1.26*10}^{-4}\text{ (Norway)}$$

$$g(I_0 - I_f) - g\frac{dy}{dx} = U\frac{dU}{dx} + \frac{dU}{dt}$$

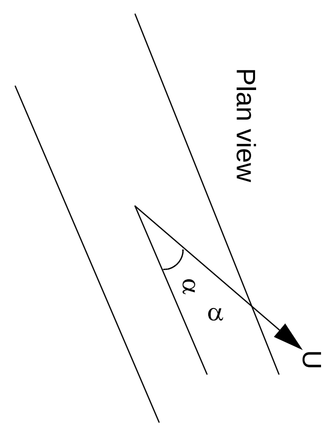
$$\tau = c_{10}\rho_a U_a^2 \qquad \Gamma = 0.11 u_* y$$

$$U_i\frac{\partial c}{\partial x_i} = \frac{\partial}{\partial x_i}\left(\Gamma\frac{\partial c}{\partial x_i}\right) - kc \qquad \Gamma = 0.058\frac{Q}{IB}$$

$$\frac{\partial U_i}{\partial t} + U_j\frac{\partial U_i}{\partial x_j} = \frac{1}{\rho}\frac{\partial}{\partial x_j}(-P\delta_{ij} - \rho\overline{u_i u_j})$$

$$-\rho\overline{u_i u_j} = \rho\nu_T\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) - \frac{2}{3}\rho k\delta_{ij}$$

$$P_k = \nu_T\frac{\partial U_j}{\partial x_i}\left(\frac{\partial U_j}{\partial x_i} + \frac{\partial U_i}{\partial x_j}\right)$$



$$K = -\frac{\sin\phi\sin\alpha}{\tan\theta} + \sqrt{\left(\frac{\sin\phi\sin\alpha}{\tan\theta}\right) + cos^2\phi\left[1 - \left(\frac{\tan\phi}{\tan\theta}\right)^2\right]}$$

$$\frac{\Delta}{h} = 0.11\left(\frac{D_{50}}{h}\right)^{0.3}\left(1 - e^{-\left[\frac{\tau-\tau_c}{2\tau_c}\right]}\right)\left(25 - \left[\frac{\tau-\tau_c}{\tau_c}\right]\right) \qquad k_s = 3D_{90} + 1.1\Delta\left(1 - e^{\frac{-25\Delta}{\lambda}}\right)$$

$$c_{10} = 1.1*10^{-3} \qquad \rho_a = 1.2\ \text{kg/m}^3 \qquad \lambda = 7.3h$$

$$\frac{c(y)}{c_a} = \left(\frac{h-y}{y}\frac{a}{h-a}\right)^z \qquad z = \frac{w}{\kappa u*} \qquad \tau = \rho g y I \qquad c = U \pm \sqrt{gy}$$

$$c = \frac{5}{3}U \qquad \rho_s = 2650\ \text{kg/m}^3 \qquad \rho_w = 1000\ \text{kg/m}^3$$

$$Fr = \frac{U}{\sqrt{gh}} \qquad Fr' = \frac{u_0}{\sqrt{\left(\frac{\rho_{res} - \rho_0}{\rho_{res}}\right) g d_0}} \qquad U = C\sqrt{Ir_h}$$

$$\frac{u}{u_0} = 4.3 Fr'^{-\frac{2}{3}} \left(\frac{z}{d_0}\right)^{-\frac{1}{3}} e^{\left[-96\frac{r^2}{z^2}\right]} \qquad \frac{\rho - \rho_{res}}{\rho_0} = 9 Fr'^{-\frac{2}{3}} \left(\frac{z}{d_0}\right)^{-\frac{5}{3}} e^{\left[-71\frac{r^2}{z^2}\right]}$$

$$\frac{Q}{Q_0} = 0.18 Fr'^{-\frac{2}{3}} \left(\frac{z}{d_0}\right)^{\frac{5}{3}} \qquad c(x, t) = \frac{c_0 L}{2\sqrt{\pi \Gamma t}} e^{-\frac{(x - Ut)^2}{4\Gamma t}}$$

$$GCI_{21} = \frac{1.25 e_{21}}{{r_{21}}^{p} - 1} \qquad e_{21} = \left|\frac{\varphi_1 - \varphi_2}{\varphi_1}\right| \qquad r_{21} = \frac{h_2}{h_1} \qquad h = \left(\frac{1}{M} \sum_{j=1}^{M} V_j\right)^{\frac{1}{3}}$$

$$R^* = \frac{u_* d}{\nu} = \frac{d\sqrt{\frac{\tau}{\rho}}}{\nu} \qquad q_s = 0.05 \rho_s U^2 \sqrt{\frac{d_{50}}{g\left(\frac{\rho_s}{\rho_w} - 1\right)}} \left[\frac{\tau}{g(\rho_s - \rho_w) d_{50}}\right]^{\frac{3}{2}}$$

$$q_s = \frac{1}{g} \left[\frac{\rho_w g r I - 0.047 g(\rho_s - \rho_w) d_{50}}{0.25 \rho_w^{\frac{1}{3}} \left(\frac{\rho_s - \rho_w}{\rho_s}\right)^{\frac{2}{3}}}\right]^{\frac{3}{2}}$$

k_3 =1.2

$$\frac{u}{u_{max}} = \left(1 + \frac{r^2}{0.016 x^2}\right)^{-2} \qquad \frac{u_{max}}{u_0} = 6.4\left(\frac{x}{d_0}\right)^{-1} \qquad \frac{Q}{Q_0} = 0.42 \frac{x}{d_0}$$

$$\frac{dy}{dx} = \frac{I_f - I_0}{1 - Fr^2} \qquad I' = -I \frac{\rho_1}{\rho'}$$

Hydrogen: 1
Carbon: 12
Nitrogen: 14
Oxygen: 16

$$d = k_1 \sqrt{\frac{q}{N}} \qquad N^2 = -\frac{g}{\rho}\frac{d\rho}{dz} \qquad d = k_3 \left(\frac{Q^2}{B^2 h N^2} \right)^{\frac{1}{3}}$$

Fall velocity of quartz spheres in water.

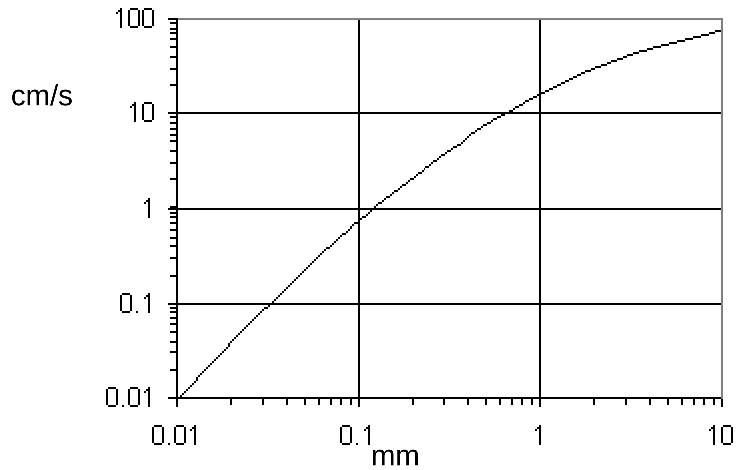
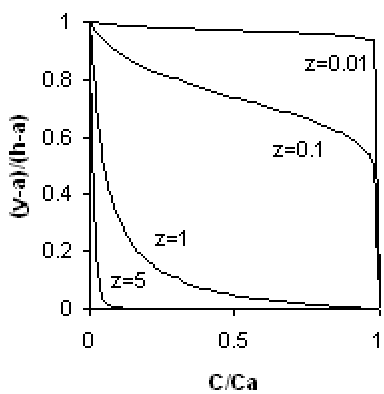


Table of water density as a function of temperature:

Temp (^{0}C)	Density (kg/m^3)
0	999.87
2	999.97
4	1000.0
6	999.97
8	999.88
10	999.73
12	999.52
14	999.27
16	998.97
18	998.62
20	998.23
22	997.80
24	997.33
26	996.81
28	996.26
30	995.68



Solution:

Problem 1.

a) U=My^2/3 I^0.5 = 60(1/300)^0.5y^0.5 = 3.46 y^2/3
q=Uy = 2m2/s
y= 0.72 m, U=2.8 m/s

b) Bed shear stress: rho g y I = 1000 kg/m * 9.81 m/s2 * 0.72 m / 300 = 23.5 Pa

Engelund-Hansens formula for total load:

$$q_s = 0.05x2650x2.8^2\sqrt{\frac{0.001}{9.81\left(\frac{2650}{1000}-1\right)}}\left[\frac{23.5}{9.81(2650-1000)0.001}\right]^{\frac{3}{2}} = \underline{\underline{14.1\ \mathrm{kg/m/s}}}$$

c)

$$\frac{dy}{dx} = \frac{I_f - I_0}{1 - Fr^2}$$

d) dx = 50 m, I0 = 1/300. Fr = U/sqrt(gy)

Guessing dy: dy = -1 m gives average y = 1.5 meter, giving average U = 1.33 m, giving Fr = 0.35 and If = 0.00287. The formula then gives

dy = 300 * (0.00287 - 1/300)/(1-0.35^2) = -1.04 m

if dy = -1.04 meter, then y is 2 m - 1.04 m = 0.96 meter. If we iterate one more time, we get the same solution, which is then correct.

Problem 2:

a) First order: $\frac{dc}{dt} = -k$

b) $6CO_2 + 6H_2O \rightarrow C_6H_{12}O_6 + 6O_2$

c)

Molecular weight of CO_2 is 44, O_2 is 32. 100 gram CO_2 will then give 32/44 * 100 = 73 grams O_2

Problem 3

a) List of errors and uncertanties

1. Modelling errors
2. Errors in the numerical approximations
3. Errors due to not complete convergence
4. Round-off errors
5. Errors in boundary conditions and input data
6. Human errors due to inexperience of the user
7. Bugs in the software

Evaluation

1. Use different models or make sure you are using the correct one
2. Parameter tests on different discretization schemes, grids etc.
3. Change the convergence criteria and see how much the results change
4. Make sure double precision is used in the computer program
5. Parameter tests on boundary conditions and input data
6. Training, making sure user has documented experience with the program, tests against measurements.
7. User more than one program

Higher-order schemes:

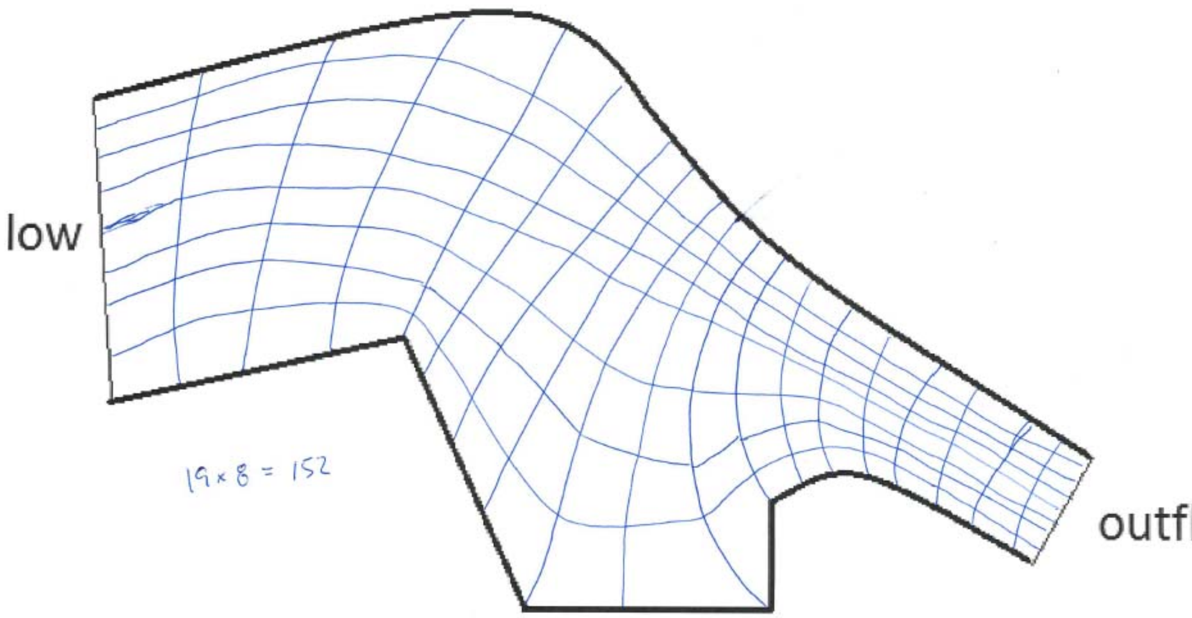
Yes, has also false diffusion, just less. The higher-order schemes will also include numerical approximations, they are just more accurate.

b)

$$GCI_{21} = \frac{1.25e_{21}}{r_{21}{}^{p} - 1} \qquad e_{21} = \left|\frac{\varphi_1 - \varphi_2}{\varphi_1}\right| \qquad r_{21} = \frac{h_2}{h_1} \qquad h = \left(\frac{1}{M}\sum_{j=1}^{M} V_j\right)^{\frac{1}{3}}$$

Table 2: Number of cells in the grids and coefficients of discharge (C) for the parameter sensitivity tests

Spillway	Fine grid 1 size	C_1	Coarse grid 2 size	C_2	r_{21}	e_{21}	GCI_{21}
Alpha	8 000 000	1.73	1 000 000	1.70	2	1.73 %	2.2 %
Beta	2 000 000	1.72	1 000 000	1.75	1.26	1.74 %	8.4 %
Delta	1 100 000	1.81	1 000 000	1.80	1.0322	0.55 %	21 %



Problem 5

a) The shear stress on the lake is computed from

$$\tau = c_{10}\rho_a U_a^2 = 1.1x10^{-3}x1.2x20^2 = 0.53Pa$$

The slope of the water surface is given by

$$I = \frac{\tau}{\rho g h} = \frac{0.53}{1000x9.81x100} = 5.4x10^{-7}$$

b) The density at 14 ^{o}C is approximately 999.27 kg/m^3 according to the table, and the density for 6 ^{o}C is 999.97 kg/m^3.

The slope of the thermocline is given by:

$$I' = -I\frac{\rho_1}{\rho'} = -5.4x10^{-7}x\frac{1000}{999.97 - 999.27} = -7.7x10^{-4}$$

c) The force from the wind is given as:

$$F_w = \tau L \tag{7.4.1}$$

The force from the hydrostatic pressure difference is:

$$F_{pd} = \frac{1}{2}\rho g H_l^2 - \frac{1}{2}\rho g H_r^2 = \rho g \frac{(H_r + H_l)}{2}(H_l - H_r) \tag{7.4.2}$$

We define the average water depth as:

$$H = \frac{(H_r + H_l)}{2} \tag{7.4.3}$$

The surface slope is given as:

$$I = \frac{H_r - H_l}{L} = -\frac{(H_l - H_r)}{L} \tag{7.4.4}$$

Combining this equation with Eq. 7.4.3 and Eq. 7.4.2, we obtain:

$$F_{pd} = -\rho g H L I \tag{7.4.5}$$

Setting the sum of the forces (Eq. 7.4.1 and 7.4.5) in the horizontal direction to zero, gives the following equation for the water surface slope:

$$I = \frac{\tau}{\rho g H}$$

Problem 6

a) Velocities: Navier-Stokes equations, k-epsilon turbulence model, SIMPLE metod for pressure, logarithmic wall laws at the boundary, continuity equation

Free surface: Pressure based method

Sediment transport: Convection-diffusion equation + van Rijn or Engelund-Hansen formula for bed concentration, continuity equation for bed level changes.

b) 105 days. Computational time: two hours on a desktop computer. This information was only given in class and it was not in the paper. The grading was therefore only done on the basis of the answer to how long the time series was for the computations.

c) Implicit solver with long time steps. Variable time step that changed according to the water discharge.

d) 12 parameters were investigated, as given in the table below:

Method	Deposits [m³]	Deviation, reference [%]	Deviation, measured [%]
Measured	51 000	-10	0.0
Computed, reference case	56 850	0.0	11
Computed, 300 seconds fixed time step (30 000 time steps)	54 721	-3.7	7.3
Computed, fine grid and 300 seconds fixed time step	50 031	-14	-1.9
Computed, with initial bed grain size distribution from reference cae	56 239	-1.1	*10*
Computed, fall velocities from Winterwerp formula for 3 finest fractions	45 305	-20	-13
Computed, 5 x larger fall velocities for 3 finest fractions	83 961	48	65
Computed, roughness from 2 to 5 cm	47 769	-16	-6.7
Computed, roughness from 2 cm to using vanRijns formula	51 947	-8.6	1.8
Computed, active layer thickness from 0.1 to 0.2 m	57 635	1.4	13
Computed, active layer thickness from 0.1 to 0.02 m	57 144	0.5	12
Computed, Engelund-Hansens formula	46 140	-19	-11
Computed, van Rijns suspended load formula only	60 458	6.3	19
Computed, first-order scheme instead of second order scheme	59 616	4.9	17
Computed, cohesion equivalent to 1 Pa on four finest fractions	70 395	24	38
Computed, cohesion equivalent to 0.1 Pa on four finest fractions	67 022	18	31
Computed, cohesion equivalent to 0.01 Pa on four finest fractions	58 264	2.5	14

e) Fall velocities (flocculation), cohesion and sediment transport formula affected the results most.